

An Improved Weight-coded Evolutionary Algorithm for the Multidimensional Knapsack Problem

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Abstract

An improved weight-coded evolutionary algorithm (IWCEA) is proposed for solving multidimensional knapsack problems. This IWCEA uses a new decoding method and incorporates a heuristic method in initialization. Computational results show that the IWCEA runs faster and performs better than a weight-coded evolutionary algorithm proposed by Raidl (1999) and to some existing benchmarks, it can yield better results than the ones reported in the OR-library.

Key words: Weight-coding, evolutionary algorithm, multidimensional knapsack problem (MKP)

1 Introduction

The multidimensional knapsack problem (MKP) can be stated as:

$$\max f(x) = \sum_{j=1}^n p_j x_j, \quad (1a)$$

$$\text{s.t. } \sum_{j=1}^n r_{ij} x_j \leq b_i, \quad i = 1, \dots, m, \quad (1b)$$

$$x_j \in \{0, 1\}, \quad j = 1, \dots, n. \quad (1c)$$

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Each of the m constraints described in (1b) is called a *knapsack* constraint. A set of n items with profits $p_j > 0$ and m resources with $b_i > 0$ are given. Each item j consumes an amount $r_{ij} \geq 0$ from each resource i . The 0-1 decision variables x_j indicate which items are selected. A *well-stated* MKP also assumes that $r_{ij} \leq b_i < \sum_{j=1}^n r_{ij}$ and $p_j > 0$ for all $i \in I = \{1, \dots, m\}$, $j \in J = \{1, \dots, n\}$, since any violation of these conditions will result in some constraints being eliminated or some x_j 's being fixed.

The MKP degenerates to the *knapsack problem* when $m = 1$ in Eq. (1b). It is well known that the knapsack problem is not a strong \mathcal{NP} -hard problem and solvable in pseudo-polynomial time. However, the situation is different to the general case of $m > 1$. Garey and Johnson (1979) [1] proved that it is strongly \mathcal{NP} -hard and exact techniques are in practice only applicable to instances of small to moderate size.

Many practical problems such as the capital budgeting problem [2], allocating processors and databases in a distributed computer system [3], project selection and cargo loading [4], and cutting stock problems [5] can be formulated as an MKP. The MKP is also a subproblem of many general integer programs.

Given the practical and theoretical importance of the MKP, a large number of papers have devoted to the problem. It is not the place here to recall all of these papers. We refer to the papers of Chu and Beasley (1998) [6], Fréville (2004) [7] and the monograph of Kellerer (2004) [8] for excellent overviews of theoretical analysis, exact methods, and heuristics of the MKP. Recently, some new algorithms for the MKP have been proposed such as some variants of the genetic algorithm [9], the ant colony algorithm [10], the scatter search method [11], and some new heuristics [12–15]. Some studies on analysis of the MKP [16, 17] and generalizations of the MKP [18–20] have also been put forward.

An *Evolutionary algorithm* (EA) is a generic population-based metaheuristic optimization algorithm. Candidate solutions to the optimization problem play the role of individuals (parents) in a population. Some mechanisms inspired by biological evolution: selection, crossover and mutation are used. The fitness function determines the environment within which the solutions “survive”. Then new groups of the population (children) are generated after the repeated application of the above operators.

In the last two decades EAs were studied for solving the MKP. Although the early works do not successfully show that *genetic algorithms* (GAs) were an effective tool for the MKP, the first successful GA's implementation was proposed by Chu and Beasley (1998) [6]. Extended numerical comparisons with CPLEX (version 4.0) and other heuristic methods showed that Chu and Beasley's GA has a robust behavior and can obtain high-quality solutions

within a reasonable amount of computational time. Raidl and Gottlieb (2005) [17] introduced and compared six different EAs for the MKP, and performed static and dynamic analyses explaining the success or failure of these algorithms, respectively. They concluded that an EA based on direct representation, combined with local heuristic improvement (referred to as DIH in [17], i.e., GA of Chu and Beasley (1998) [6] with slight revision), can achieve better performance than other EAs mentioned in [17] from empirical analysis.

The best success for solving the MKP, as far as we known, has been obtained with tabu-search algorithms embedding effective preprocessing [21, 22]. Recently, impressive results have also been obtained by an implicit enumeration [23], a convergent algorithm [24], and an exact method based on a multi-level search strategy [25]. Compared with EAs, the methods mentioned above can yield better results when excellent solutions are required. But they are more complicated to implement or their computation takes extremely long time. Since EAs are simple to implement and their computation time are easy to control, they are good alternatives if the quality requirement of solutions of the MKP is not very strict.

In this paper, we will consider a variant of EA to solve the MKP. This EA will use a special encoding technique which is called *weight-coding* (or *weight-biasing*). We will improve a weight-coded EA (WCEA) proposed by Raidl (1999) [26] and propose an improved weight-coded EA (IWCEA). The numerical experiments of some benchmarks will show that the IWCEA performs better than the WCEA and can compete with DIH in some benchmarks. Moreover, in the same platform, IWCEA's iterate time is shorter than other EAs listed in [17].

2 An Introduction to the weight-coding and its application to the MKP

When combinatorial optimization problems are solved by an EA, the coding of candidate solutions is a preliminary step. Direct coding such as the *binary coding* is an intuitive method. The main drawback of this coding lies in that many infeasible solutions may be generated by EA's operators. To avoid that, the basic idea of the weight-coding is to represent a candidate solution by a vector of real-valued weights w_j ($j = 1, \dots, n$). The *phenotype* that a weight vector represents is obtained by a two-step process.

- Step (a): (*biasing*) The original problem P is temporarily modified to P' by biasing problem parameters of P according to the weights w_j ;
- Step (b): (*decoding heuristic*) A problem-specific decoding heuristic is used to generate a solution to P' . This solution is interpreted and evaluated

for the original (unbiased) problem P .

The weight-coding is an interesting approach because it can eliminate the necessity of an explicit repair algorithm, a penalization of infeasible solutions, or special crossover and mutation operators. It has already been successfully used for a variety of problems such as an optimum communications spanning tree problem [27], problem [28], the traveling salesman problem [29], and the multiple container packing problem [30].

To the best of the authors' knowledge, the work of Raidl (1999) [26] is the first to use weight-coded EA (WCEA) to deal with the MKP. In that paper, some variants of WCEAs were proposed and compared. And Raidl finally suggested one of them and compared the WCEA with other EAs in [17]. In this WCEA, w_j ($j = 1, \dots, n$) is set to be the weight vector representing a candidate solution. Weight w_j is associated with item j of the MKP. Corresponding to Step (a), the original MKP is biased by multiplying of profits in (1a) with *log-normally* distributed weights:

$$p'_j = p_j w_j = p_j (1 + \gamma)^{\mathcal{N}(0,1)}, \quad j = 1, \dots, n \quad (2)$$

where $\mathcal{N}(0,1)$ denotes a normally distributed random number with mean 0 and standard deviation 1, and $\gamma > 0$ is a strategy parameter that controls the average intensity of biasing. Raidl (1999) [26] suggested that $\gamma = 0.05$. Since the resource consumption values r_{ij} and resource limits b_i are not modified, all feasible solutions of the biased MKP are feasible to (1).

Corresponding to Step (b), the decoding heuristic which Raidl (1999) [26] suggested is making use of the *surrogate relaxation* (See [31, 32]). The m resource constraints (1b) are aggregated into a single constraint using surrogate multipliers a_i , $i = 1, \dots, m$:

$$\sum_{j=1}^n \left(\sum_{i=1}^m a_i r_{ij} \right) x_j \leq \sum_{i=1}^m a_i b_i \quad (3)$$

where a_i are obtained by solving the linear programming (LP) of the relaxed MKP, in which the variables x_j may get real values from $[0, 1]$. The values of the dual variables are then used as surrogate multipliers, i.e. a_i is set to the shadow price of the i -th constraint in the LP-relaxed MKP. *Pseudo-utility ratios* are defined as:

$$u_j = \frac{p'_j}{\sum_{i=1}^m a_i r_{ij}}. \quad (4)$$

A higher pseudo-utility ratio heuristically indicates that an item is more efficient. After the items are sorted by decreasing order of u_j , the *first-fit strategy* used as decoder in the permutation representation is applied. All items are checked one by one and each item's variable x_j is set to 1 if no resource constraint is violated, otherwise, x_j is set to 0. The computational effort of the

decoder is $O(n \cdot \log n)$ for sorting the u_j plus $O(n \cdot m)$ for the first-fit strategy, yielding $O(n \cdot (m + \log n))$ in total.

Raidl's WCEA can be described as follows (we will explain the details of Steps 6, 7, and 8 afterward):

Algorithm of Raidl's WCEA

- Step 1: set $t := 0$;
- Step 2: initialize $pop(t) = \{S_1, \dots, S_N\}$, $S_i = (w_1, \dots, w_n)$ where w_j is a random value following log-normally distribution as (2);
- Step 3: evaluate $pop(t) : \{f(S_1), \dots, f(S_N)\}$;
 for each S_i
 3-1: bias original MKP;
 3-2: use decoding heuristic as in [26] (described above) to get phenotype $\mathcal{P}(S_i) \in \{0, 1\}^n$;
 3-3: substitute $\mathcal{P}(S_i)$ into (1a) to obtain $f(S_i)$;
- Step 4: find $S^* \in pop(t)$ s.t. $f(S^*) \geq f(S)$, $\forall S \in pop(t)$; $t < t_{\max}$ **do**
- Step 5: select $\{p_1, p_2\}$ from $pop(t)$;
- Step 6: crossover p_1 and p_2 to generate a child C ;
- Step 7: mutate C ;
- Step 8: evaluate C as Step 3, get $\mathcal{P}(C)$ and $f(C)$;
- Step 9: **if** $\mathcal{P}(C) \equiv \text{any } \mathcal{P}(S_i)$ **then** (that means C is a duplicate of a member of the population)
- Step 10: discard C and goto Step 6;
 end if
- Step 11: find $S' \in pop(t)$ s.t. $f(S') \leq f(S) \forall S \in pop(t)$ and replace $S' \leftarrow C$;
 (*steady-state replacement*, i.e., the worst individual of population is replaced.)
- Step 12: **if** $f(C) > f(S^*)$ **then**
- Step 13: $S^* \leftarrow C$; (update best solution S^* found)
 end if
- Step 14: $t \leftarrow t + 1$;
 end while
- Step 15: return $S^*, f(S^*)$.

In Step 6, a *binary tournament selection* is used. That is, two pools of individuals, which consist of 2 individuals drawn from the population randomly, are formed respectively at first. Then two individuals with the best fitness, each taken from one of the two tournament pools, are chosen to be parents.

In Step 7, Raidl (1999) [26] suggested a *uniform crossover* instead of one- or two-point crossover. In the uniform crossover two parents have one child. Each $w_j (j = 1, \dots, n)$ in the child is chosen randomly by copying the corresponding weight from one or the other parent.

Once a child has been generated through the crossover, a *mutation* step in Step 8 is performed. Each w_j of the child is reset to a new random value observing log-normal distribution with a small probability ($3/n$ per weight as in [26] or one random position in [17]).

In numerical experiments, the N in Step 2 is taken as 100 and t_{\max} in Step 5 is taken 10^6 . Raidl and Gottlieb (2005) [17] compared this WCEA with other five EAs for the MKP. From empirical analysis, this WCEA outperformed all of them except DIH (The meaning of DIH is given in Section 1) on average.

3 An Improved WCEA for the MKP

3.1 Motivation

The core of Raidl’s WCEA is the surrogate relaxation based heuristic in decoding. In our points of view, this heuristic has two drawbacks. First, the dual variables of an LP-relaxed MKP used in heuristic decoding step are just good approximations of optimal surrogate multipliers and it may mislead the search [21]. LP-relaxed MKP used in heuristic decoding step are just approximations of optimal surrogate multipliers. And deriving optimal surrogate multipliers is a difficult task in practice [33]. Secondly, the heuristic decoding might mislead the search if the optimal solution is not very similar to the solution generated by applying the greedy heuristic [34].

In order to avoid using surrogate multipliers, we set w_j ($j = 1, \dots, n$) to let every w_j observe uniform distribution on $[0, p_{\max}/p_j]$, where $p_{\max} = \max\{p_j : j = 1, \dots, n\}$. The profits of the original MKP are biased by multiplying weights:

$$p'_j = p_j w_j, \quad j = 1, \dots, n. \quad (5)$$

as mentioned in Section II, all feasible solutions of this biased MKP are feasible to (1). In decoding heuristic, we also use first-fit strategy, i.e., the items are sorted by decreasing order of p'_j (not by pseudo-utility ratio in (4)) and traversed. Each item’s variable x_j is set to 1 if no resource constraint is violated. The computational effort of the decoder is also $O(n \cdot (m + \log n))$ in total.

This form of w_j is similar to the idea of *Random-key Representation* [35]. Surrogate multipliers can be avoided but the efficiency of the EA will be reduced [17]. To overcome this disadvantage, our thought is to obtain a “good” initial population. In the following we first introduce an idea proposed by Vasquez and Hao [21] and then propose our method.

It is well known that only relaxing the integrality constraints in an MKP may not be sufficient because its optimal solution may be far away from the optimal binary solution. However, Vasquez and Hao in [21] observed when the integrality constraints was replaced by a *hyperplane constraint* $\sum_{j=1}^n x_j = k \in \mathbb{N}$, the corresponding linear programming solution may often be close to the optimal binary solution. For example in [21], in (1) we let $n = 5$, $m = 1$, $\mathbf{p} = \{12, 12, 9, 8, 8\}$, $\mathbf{r} = \{11, 12, 10, 10, 10\}$, $b = 30$. The relax linear programming problem leads to the fractional optimal solution $x^{LP} = \{1, 1, 0.7, 0, 0\}$ while the optimal binary solution is $x = \{0, 0, 1, 1, 1\}$. If we replace the integrality constraints by $\sum_{j=1}^n x_j = 3$, this linear programming problem leads to the optimal binary solution.

In the above example, if we take $\mathbf{w} = \{0, 0, 1, 1, 1\}$ and substitute it to (5), the optimal binary solution can be obtained by first-fit heuristic mentioned above. Moreover, if we do not restrict k as an integer, we may also obtain some corresponding linear programming solutions from which some good binary solutions may be obtained by first-fit heuristic. We use these linear programming solutions as a “good” initial population. So the disadvantage of Random-key Representation may be overcome. The experimental results presented later have confirmed this hypothesis. Naturally, the hypothesis does not exclude the possibility that there exists a certain MKP whose optimal binary solution cannot be obtained from linear programming solutions.

Inspired by this idea, initialization is guided by the LP relaxation with a hyperplane constraint. To begin with, we use some simple heuristic (such as a greedy algorithm) to obtain a 0-1 lower bound z . Next, the two following problems:

$$\begin{aligned}
k_{\max} &= \max \sum_{j=1}^n x_j, \\
\text{s.t. } \sum_{j=1}^n r_{ij} x_j &\leq b_i, & i = 1, \dots, m, \\
\sum_{j=1}^n p_j x_j &\geq z + 1 \\
x_j &\in [0, 1] & j = 1, \dots, n
\end{aligned}$$

and

$$\begin{aligned}
k_{\min} &= \min \sum_{j=1}^n x_j, \\
\text{s.t. } \sum_{j=1}^n r_{ij} x_j &\leq b_i, & i = 1, \dots, m, \\
\sum_{j=1}^n p_j x_j &\geq z + 1 \\
x_j &\in [0, 1] & j = 1, \dots, n
\end{aligned}$$

are solved to obtain k_{\max} and k_{\min} .

Then, N linear programming problems

$$\begin{aligned}
\max \sum_{j=1}^n p_j x_j, \\
\text{s.t. } \sum_{j=1}^n r_{ij} x_j &\leq b_i, & i = 1, \dots, m, \\
\sum_{j=1}^n x_j &= k' \\
x_j &\in [0, 1] & j = 1, \dots, n
\end{aligned}$$

are solved where k' is a real number generated randomly from $[k_{\min}, k_{\max}]$ in each computation. So the N linear programming solutions are generated as the initial population.

3.2 Implementation

The scheme of the IWCEA is similar to Raidl's WCEA. And we take the same values of N and t_{\max} as the WCEA. The differences between the two algorithms lie in the following aspects:

- (1) Each w_j in Raidl's WCEA observes log-normal distribution, while in IWCEA it observes a uniform distribution on $[0, p_{\max}/p_j]$, where $p_{\max} = \max\{p_j : j = 1, \dots, n\}$;
- (2) Raidl's WCEA sorts items by pseudo-utility ratios in heuristic decoding step while the IWCEA sorts items by biased profits directly;
- (3) The initial population in Raidl's WCEA is generated randomly, while in the IWCEA, N linear programming problems should be solved;
- (4) In the mutation step, one random w_j of the child is reset to a new random value observing uniform distribution on $[0, p_{\max}/p_j]$ instead of log-normal distribution in the IWCEA.

4 Experimental comparison

We use two test suites of MKP’s benchmark instances for experimental comparison. The first one, referred to as CB-suite in this paper, is introduced by Chu and Beasley (1998) [6] and is available in the OR-Library¹. This test suite contains 270 instances for each 10 ones are combination of $m \in \{5, 10, 30\}$ constraints, $n \in \{100, 250, 500\}$ items, and tightness ratio $\alpha \in \{0.25, 0.5, 0.75\}$. Each problem has been generated randomly such that $b_i = \alpha \cdot \sum_{j=1}^n r_{ij}$ for all $i = 1, \dots, m$. Chu and Beasley used their GA (i.e., DIH) to solve these instances and reported their results in the OR-library. The second MKP’s benchmark suite² used in [17] was first referenced by [21] and originally provided by Glover and Kochenberger. These instances, called GK01 to GK11, range from 100 to 2500 items and from 15 to 100 constraints. We call this suite GK-suite in this paper.

Although some commercial integral linear programming (ILP) solvers, such as **Cplex**, can solve ILP problems with thousands of integer variables or even more, it seems that the MKP remains rather difficult to handle when an optimal solution is wanted. To CB-suit, the results in [6] showed that major instances of this suit cannot be solved in a reasonable amount of CPU time and memory by **Cplex**. To GK-suit, which includes still more difficult instances with n up to 2500, Fréville (2004) in [7] mentioned that **Cplex** cannot tackle these instances. Therefore, it appears that the MKP continues to be a challenging problem for commercial ILP solvers.

The best known solutions to these benchmarks, as far as we known, were obtained by Vasquez and Hao (2001) [21] and was improved by Vasquez and Vimont (2005) [22]. Their method is based on tabu search and time-consuming compared with EA.

Raidl and Gottlieb (2005) [17] tested six different variants of EAs, which are called Permutation Representation (PE), Ordinal Representation (OR), Random-Key Representation (RK), Weight-Biased Representation (WB), i.e. Raidl’s WCEA, and Direct Representation (DI and DIH). We compare the IWCEA with these EAs except DIH first. We use all GK-suite and draw out nine instances (called CB1 to CB9) from CB-suite, which are the first instances with $\alpha = 0.5$ for each combination of m and n .

For a solution x , the *gap* is defined as:

$$gap = \frac{f(x^{LP}) - f(x)}{f(x^{LP})}$$

¹ <http://people.brunel.ac.uk/~mastjjb/jeb/info.html>

² This suite can be downloaded from <http://hces.bus.olemiss.edu/tools.html>

instance			gap[%](and standard deviation)						
name	m	n	PE	OR	RK	DI	WB	DIH	IWCEA
CB1	5	100	0.425 (0.000)	0.745 (0.210)	0.425 (0.000)	0.425 (0.000)	0.425 (0.000)	0.425 (0.000)	0.425 (0.000)
CB2	5	250	0.120 (0.012)	1.321 (0.346)	0.115 (0.009)	0.150 (0.019)	0.106 (0.007)	0.106 (0.006)	0.112 (0.007)
CB3	5	500	0.081 (0.016)	2.382 (0.657)	0.065 (0.010)	0.121 (0.020)	0.042 (0.008)	0.038 (0.003)	0.036 (0.004)
CB4	10	100	0.762 (0.001)	1.013 (0.163)	0.762 (0.003)	0.770 (0.013)	0.761 (0.000)	0.762 (0.003)	0.762 (0.003)
CB5	10	250	0.295 (0.033)	1.498 (0.225)	0.277 (0.021)	0.324 (0.043)	0.249 (0.017)	0.261 (0.008)	0.271 (0.014)
CB6	10	500	0.225 (0.040)	2.815 (0.462)	0.200 (0.029)	0.263 (0.040)	0.131 (0.014)	0.112 (0.007)	0.108 (0.002)
CB7	30	100	1.372 (0.134)	1.800 (0.182)	1.338 (0.123)	1.401 (0.073)	1.319 (0.093)	1.336 (0.091)	1.276 (0.077)
CB8	30	250	0.608 (0.048)	2.076 (0.346)	0.611 (0.072)	0.599 (0.059)	0.535 (0.031)	0.519 (0.013)	0.525 (0.002)
CB9	30	500	0.429 (0.058)	3.267 (0.442)	0.376 (0.037)	0.463 (0.056)	0.306 (0.024)	0.288 (0.012)	0.296 (0.012)
GK01	15	100	0.377 (0.068)	0.683 (0.098)	0.384 (0.080)	0.336 (0.074)	0.308 (0.077)	0.270 (0.028)	0.325 (0.077)
GK02	25	100	0.503 (0.062)	0.959 (0.144)	0.521 (0.068)	0.564 (0.067)	0.481 (0.045)	0.460 (0.007)	0.458 (0.000)
GK03	25	150	0.517 (0.060)	1.002 (0.140)	0.531 (0.077)	0.517 (0.066)	0.452 (0.042)	0.366 (0.007)	0.374 (0.034)
GK04	50	150	0.712 (0.090)	1.164 (0.143)	0.748 (0.098)	0.706 (0.079)	0.669 (0.081)	0.528 (0.021)	0.527 (0.027)
GK05	25	200	0.462 (0.072)	1.124 (0.153)	0.552 (0.118)	0.493 (0.087)	0.397 (0.046)	0.294 (0.004)	0.289 (0.012)
GK06	50	200	0.703 (0.070)	1.236 (0.141)	0.751 (0.108)	0.714 (0.077)	0.611 (0.060)	0.429 (0.018)	0.417 (0.015)
GK07	25	500	0.523 (0.088)	1.468 (0.092)	0.651 (0.087)	0.496 (0.089)	0.382 (0.082)	0.093 (0.004)	0.111 (0.005)
GK08	50	500	0.749 (0.086)	1.517 (0.109)	0.835 (0.125)	0.749 (0.085)	0.534 (0.066)	0.166 (0.006)	0.169 (0.013)
GK09	25	1500	0.890 (0.075)	2.312 (0.113)	1.064 (0.133)	0.695 (0.070)	0.558 (0.042)	0.029 (0.001)	0.030 (0.001)
GK10	50	1500	1.101 (0.065)	1.883 (0.076)	1.177 (0.082)	0.950 (0.090)	0.727 (0.070)	0.052 (0.003)	0.053 (0.002)
GK11	100	2500	1.237 (0.060)	1.677 (0.056)	1.246 (0.067)	1.161 (0.063)	0.867 (0.061)	0.052 (0.002)	0.056 (0.002)
average			0.605 (0.057)	1.597 (0.215)	0.631 (0.068)	0.595 (0.057)	0.493 (0.043)	0.329 (0.012)	0.331 (0.015)

Table 1

Average gaps of best solutions and their standard deviations of the IWCEA and other EAs

where x^{LP} is the optimum of the LP-relaxed problem to measure the quality of x .

We implement the IWCEA on a personal computer (Inter Core™ Duo T5800, 2 GHz, 1.99 GB main memory, Windows XP) using DEV-C++. The initial population is generated by MATLAB. The population size is 100, and each run was terminated after 10^6 created solution candidates; rejected duplicates were not counted.

Table 1 shows the average gaps of the final solutions and their standard deviations obtained from independent 30 runs per problem instance obtained by the IWCEA and other six variants. The results of other six variants come from [17]. The results in Table 1 show that the IWCEA outperformed PE, OR, RK, and DI. On all instances but CB2, CB4, CB5, and GK01, the IWCEA performed equal or better than Raidl’s WCEA. Especially in GK02 to GK11, the IWCEA performed much better than Raidl’s method.

Table 1 also shows that the IWCEA performed averagely slightly worse than DIH. But we will point out that can yield better results than DIH in some instances. Since the best results can be obtained by CPLEX in CB-suite when $\{m, n\} = \{5, 100\}$, $\{10, 100\}$, and $\{5, 250\}$, we tested the other 180 instances in CB-suite. Each instance was computed 30 times and the best results were compared with the results reported in OR-library. The statistical data of the numbers that the IWCEA yielded better, equal or worse results than the results reported in OR-library is shown in Table 2. Tables 3 to 8 show the comparison of each instance. These tables show that the results of more than 50% instances can be improved by the IWCEA.

m	n	number of the instance	better	equal	worse
30	100	30	2	28	0
10	250	30	12	16	2
30	250	30	15	10	5
5	500	30	19	9	2
10	500	30	23	4	3
30	500	30	21	4	5
Total		180	92	71	17

Table 2

The statistical data of the numbers that the IWCEA yielded better, equal and worse results than the results reported in OR-library

5 Conclusion

We have proposed an IWCEA for solving multidimensional knapsack problems. This IWCEA has been different from Raidl's WCEA in the ways that surrogate multipliers are not used and a heuristic method is incorporated in initialization. Experimental comparison has shown that the IWCEA can yield better results than Raidl's WCEA in [26] and better results than the ones reported in the OR-library to some existing benchmarks.

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CB	OR _{CB}	IWCEA	CB	OR _{CB}	IWCEA
30.100.00	21946	21946	30.100.15	41058	41058
30.100.01	21716	21716	30.100.16	41062	41062
30.100.02	20754	20754	30.100.17	42719	42719
30.100.03	21464	21464	30.100.18	42230	42230
30.100.04	21814	21814	30.100.19	41700	41700
30.100.05	22176	22716	30.100.20	57494	57494
30.100.06	21799	21799	30.100.21	60027	60027
30.100.07	21397	21397	30.100.22	58025	58025
30.100.08	22493	22493	30.100.23	60776	60776
30.100.09	20983	20983	30.100.24	58884	58884
30.100.10	40767	40767	30.100.25	60011	60011
30.100.11	41304	41304	30.100.26	58132	58132
30.100.12	41560	41587	30.100.27	59064	59064
30.100.13	41041	41041	30.100.28	58975	58975
30.100.14	40872	40889	30.100.29	60603	60603

Table 3

The results of CB-suite reported in OR-library (OR_{CB}) and the ones obtained by the IWCEA ($m = 30$, $n = 100$)

CB	OR _{CB}	IWCEA	CB	OR _{CB}	IWCEA
10.250.00	59187	59187	10.250.15	110841	110841
10.250.01	58662	58708	10.250.16	106075	106075
10.250.02	58094	58094	10.250.17	106686	106686
10.250.03	61000	61000	10.250.18	109825	109825
10.250.04	58092	58092	10.250.19	106723	106723
10.250.05	58803	58803	10.250.20	151790	151801
10.250.06	58607	58704	10.250.21	147822	148772
10.250.07	58917	58930	10.250.22	151900	151900
10.250.08	59384	<i>59382</i>	10.250.23	151275	151281
10.250.09	59193	59208	10.250.24	151948	151966
10.250.10	110863	110913	10.250.25	152109	151209
10.250.11	108659	108702	10.250.26	153131	153131
10.250.12	108932	108932	10.250.27	153520	153578
10.250.13	110037	<i>110034</i>	10.250.28	149155	149160
10.250.14	108423	108485	10.250.29	149704	149704

Table 4

The results of CB-suite reported in OR-library (OR_{CB}) and the ones obtained by the IWCEA ($m = 10$, $n = 250$)

CB	OR _{CB}	IWCEA	CB	OR _{CB}	IWCEA
30.250.00	56693	56747	30.250.15	107246	<i>107183</i>
30.250.01	58318	58520	30.250.16	106308	<i>106261</i>
30.250.02	56553	56553	30.250.17	103993	103993
30.250.03	56863	56930	30.250.18	106835	<i>106800</i>
30.250.04	56629	56629	30.250.19	105751	105751
30.250.05	57119	57146	30.250.20	150083	150096
30.250.06	56292	<i>56290</i>	30.250.21	149907	149907
30.250.07	56403	56457	30.250.22	152993	153007
30.250.08	57442	<i>57429</i>	30.250.23	153169	153190
30.250.09	56447	56447	30.250.24	150287	150287
30.250.10	107689	107737	30.250.25	148544	148544
30.250.11	108338	108379	30.250.26	147471	147471
30.250.12	106385	106433	30.250.27	152841	152877
30.250.13	106796	106806	30.250.28	149568	149570
30.250.14	107396	107396	30.250.29	149572	149601

Table 5

The results of CB-suite reported in OR-library (OR_{CB}) and the ones obtained by the IWCEA ($m = 30$, $n = 250$)

CB	OR _{CB}	IWCEA	CB	OR _{CB}	IWCEA
5.500.00	120130	120145	5.500.15	220514	220520
5.500.01	117837	117864	5.500.16	219987	219989
5.500.02	121109	121118	5.500.17	218194	218215
5.500.03	120798	120798	5.500.18	216976	216976
5.500.04	122319	122319	5.500.19	219693	219719
5.500.05	122007	122009	5.500.20	295828	295828
5.500.06	119113	119127	5.500.21	308077	308083
5.500.07	120568	120568	5.500.22	299796	299796
5.500.08	121575	121575	5.500.23	306476	306480
5.500.09	120699	120717	5.500.24	300342	300342
5.500.10	218422	218428	5.500.25	302560	<i>302559</i>
5.500.11	221191	<i>221188</i>	5.500.26	301322	301329
5.500.12	217534	217542	5.500.27	296437	296457
5.500.13	223558	223560	5.500.28	306430	306454
5.500.14	218962	218966	5.500.29	299904	299904

Table 6

The results of CB-suite reported in OR-library (OR_{CB}) and the ones obtained by the IWCEA ($m = 5$, $n = 500$)

CB	OR _{CB}	IWCEA	CB	OR _{CB}	IWCEA
10.500.00	117726	117779	10.500.15	215013	215041
10.500.01	119139	119181	10.500.16	217896	217911
10.500.02	119159	119194	10.500.17	219949	219984
10.500.03	118802	<i>118784</i>	10.500.18	214332	214346
10.500.04	116434	116471	10.500.19	220833	220865
10.500.05	119454	119461	10.500.20	304344	304344
10.500.06	119749	119777	10.500.21	302332	302333
10.500.07	118288	<i>118277</i>	10.500.22	302354	302408
10.500.08	117779	<i>117750</i>	10.500.23	300743	300747
10.500.09	119125	119175	10.500.24	304344	304350
10.500.10	217318	217318	10.500.25	301730	301757
10.500.11	219022	219033	10.500.26	304949	304949
10.500.12	217772	217772	10.500.27	296437	296457
10.500.13	216802	216819	10.500.28	301313	301353
10.500.14	213809	213827	10.500.29	307014	307072

Table 7

The results of CB-suite reported in OR-library (OR_{CB}) and the ones obtained by the IWCEA ($m = 10$, $n = 500$)

CB	OR _{CB}	IWCEA	CB	OR _{CB}	IWCEA
30.500.00	115868	<i>115864</i>	30.500.15	215762	215832
30.500.01	114667	114701	30.500.16	215772	215839
30.500.02	116661	116661	30.500.17	216336	216419
30.500.03	115237	<i>115228</i>	30.500.18	217290	217302
30.500.04	116353	116370	30.500.19	214624	214634
30.500.05	115604	115639	30.500.20	301627	301643
30.500.06	113952	113983	30.500.21	299985	<i>299958</i>
30.500.07	114199	114230	30.500.22	304995	305062
30.500.08	115247	115247	30.500.23	301935	301935
30.500.09	116947	116947	30.500.24	304404	304411
30.500.10	217995	218042	30.500.25	296894	296955
30.500.11	214534	214557	30.500.26	303233	303262
30.500.12	215854	215885	30.500.27	306944	306985
30.500.13	217836	<i>217773</i>	30.500.28	303057	303120
30.500.14	215566	<i>215553</i>	30.500.29	300460	300531

Table 8

The results of CB-suite reported in OR-library (OR_{CB}) and the ones obtained by the IWCEA ($m = 30$, $n = 500$)